15.18 Comparison of the fatigue life prediction on the welded part according SBRA method and Eurocode 3

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Assignment:

Determine the high cycle fatigue life of a welded joint (located on the bucket wheel of the excavator gearwheel). Assessment according to:

a) Eurocode 3 (detail category 112), [15.18.1]

b) SBRA method (using AntHill program)

The probability of fracture and fatigue aspects are evaluated and discussed.

Input:

Service loading of the bucket wheel excavator have been monitored and registered. The scraper forces spectrum has been obtained during the long-term measurement involving approximately 1000 hours. Because of the flange of the gearwheel is submitted during its rotation also to the transient cycles, the superposition of the both periodic and stochastic loading conditions have been taken into account (see example of time signal on Fig. 1).

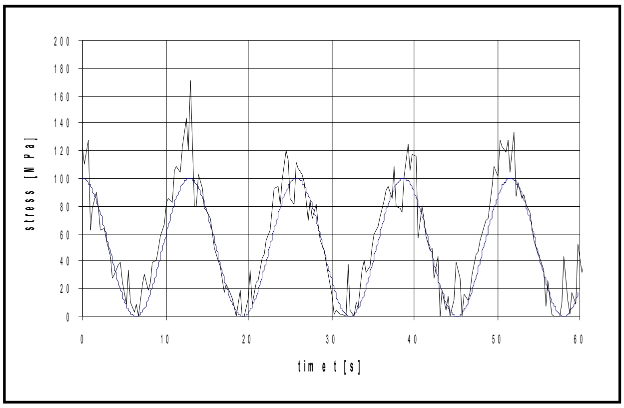

Fig.1 Stress-time signal in the weld.

Quantitative values of the equivalent nominal stress in the weld have been obtained using by FEM calculation. A generated 1000 hours quasi-dynamic stress-time signal has been utilized as an input to the "rain-flow" analysis. Results of this analysis are two-parametric matrices. Consequently, these two-parametric matrices are recalculated to the one parametric form of stress spectrum (see Table 1).

Table 1 Stress ranges and number of cycles during 1000 hours.

i	$\Delta\sigma_i$ [MPa]	n_i [cycles]
1	5.2	487132
2	14.4	680888
3	25.8	411287
4	36.0	199288
5	46.2	83025
6	56.6	30456
7	66.8	9931
8	77.2	4105
9	87.4	1854
10	97.8	795
11	108.0	265
12	118.4	10329
13	128.6	30853
14	138.8	16949
15	149.2	2781
16	159.4	530

Fatigue life assessment of the welded joint is based on the stress-life curve (*S-N* curve) depicted in Eurocode 3, detail category 112 (Double slope curve, with the constants m=3 and m=5 and the fatigue limit $\Delta\sigma_D$=83 MPa at the $N_D=5\cdot10^6$ cycles. The cut-off limit is $\Delta\sigma_L=45$ MPa at the $N_D=10^8$ cycles).

The thickness of the welded flange is t=43 mm; hence, according to [15.18.1] the reduction factor $\varphi_t = \left(\frac{25}{t}\right)^{0.2}$ have been calculated.

Comments to the assignment:

Fatigue life calculation of welded joints according the Eurocode 3 [15.18.1] is basically deterministic. The result of this approach is the safety life. However, this safety life does not allow quantify the probability of the fracture. Acceptable failure probability for most applications are in the range $P_f=10^{-3}...10^{-5}$. This probability should be guarantied using by two partial safety factors (γ_{Ff} and γ_{Mf}) with respect to the both variable: the stress range and the fatigue strength. Already published *S-N* curves for different weld categories represent "safe" curves with probability of failure about 2.3%. The analysis published in [15.18.2] present failure probability by the downshift of the *S-N* curve by two standard deviations (s=0.0688).

SBRA method uses purely stochastic approach and enables to predict fatigue life for arbitrary selected failure probability. However, the simulation has to result from *S-N* curve of 50% failure probability. Therefore in SBRA method calculation the value of the Eurocode *S-N* curve fatigue limit has been shifted to $\Delta\sigma_D$=96.2 MPa and the cut-off limit to $\Delta\sigma_L=52.2$ MPa.

Calculation background of the fatigue life:

The theoretical background of fatigue life and reliability function calculations is the same as described in example 11.7 of this book.

Fatigue damage D_b corresponding to the loading spectra (1000 hours) with stress ranges $\Delta\sigma_{ef,i}$ can be written as

$$D_b = \frac{\sum a_i}{N_D} = \frac{\sum n_i \cdot \left(\frac{\Delta\sigma_{ef,i}}{\Delta\sigma_D} \right)^m}{N_D}$$

Using by Palmgren-Miner's rule with critical damage D_M, the number of loading cycles to failure b can be written as follow:

$$b = \frac{D_M}{D_b} .$$

The reliability function $RF=R\text{-}S$ related to 1000 hours of service is expressed as

$$RF = \frac{D_M}{b} - D_b .$$

Calculation according to Eurocode 3:

All input data are revolving as constants, with respect to the Input section. Fatigue damage is summarised with respect of the three parts of the *S-N* curve, follows:

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b=DM/Db

Db=a/ND

a=a16+a15+a14+a13+a12+a11+a10+a9+a8+a7+a6+a5+a4+a3+a2+a1

a16=n16*((1-pos(sL-s16))*s16/sD/fi)^(3+2*pos(sD-s16))

a15=n15*((1-pos(sL-s15))*s15/sD/fi)^(3+2*pos(sD-s15))

a14=n14*((1-pos(sL-s14))*s14/sD/fi)^(3+2*pos(sD-s14))

a13=n13*((1-pos(sL-s13))*s13/sD/fi)^(3+2*pos(sD-s13))

a12=n12*((1-pos(sL-s12))*s12/sD/fi)^(3+2*pos(sD-s12))

a11=n11*((1-pos(sL-s11))*s11/sD/fi)^(3+2*pos(sD-s11))

a10=n10*((1-pos(sL-s10))*s10/sD/fi)^(3+2*pos(sD-s10))

a9=n9*((1-pos(sL-s9))*s9/sD/fi)^(3+2*pos(sD-s9))

a8=n8*((1-pos(sL-s8))*s8/sD/fi)^(3+2*pos(sD-s8))

a7=n7*((1-pos(sL-s7))*s7/sD/fi)^(3+2*pos(sD-s7))

a6=n6*((1-pos(sL-s6))*s6/sD/fi)^(3+2*pos(sD-s6))

a5=n5*((1-pos(sL-s5))*s5/sD/fi)^(3+2*pos(sD-s5))

a4=n4*((1-pos(sL-s4))*s4/sD/fi)^(3+2*pos(sD-s4))

a3=n3*((1-pos(sL-s3))*s3/sD/fi)^(3+2*pos(sD-s3))

a2=n2*((1-pos(sL-s2))*s2/sD/fi)^(3+2*pos(sD-s2))

a1=n1*((1-pos(sL-s1))*s1/sD/fi)^(3+2*pos(sD-s1))

fi=(25/tlred)^0.2
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tlred=pos(tl-25)*(tl-25)+25

Results:

Calculated number of repeated loading spectra is b=13.537 what represents the life of 13 537 hours. Because of the calculation described above did not involve the partial safety factors γ_{Ff} and γ_{Mf}, presented results represent the failure probability 2.3% (what corresponds to the Eurocode *S-N* curves). If safety factors γ_{Ff}=1.0 and γ_{Mf}=1.15 would be taken into account the safe fatigue life would be b=8.70 respective b=6.41 for γ_{Ff}=1.1 and γ_{Mf}=1.15.

Calculation according to SBRA method:

This method allows fatigue life prediction with the arbitrary failure probability. In presented example, variance of the loading cycle (frequency), weld flange thickness variance, fatigue limit scatter and the cut of limit in the *S-N* curve and the critical damage have been taken into account.

Variables:

Next random quantities can be proposed:

- variation of the loading cycles by lognormal distribution (mean according Table 1 and standard deviation about 10% of this value)
- geometrical scatter of the thickness by normal distribution (mean=43, st. deviation=1)
- variation of the fatigue limit and the cut of limit in the *S-N* curve (P=50%) by Normal distribution (mean=0, st. deviation=0.0688)
- critical damage by normal distribution (mean=1, st. deviation=0.1)

Calculation:

SF=R-S

R=DDM/b

S=Db

b=DM/Db

Db=a/ND

a=a16+a15+a14+a13+a12+a11+a10+a9+a8+a7+a6+a5+a4+a3+a2+a1

a16=nn16*((1-pos(sL-s16))*s16/sD/fi)^(3+2*pos(sD-s16))

a15=nn15*((1-pos(sL-s15))*s15/sD/fi)^(3+2*pos(sD-s15))

a14=nn14*((1-pos(sL-s14))*s14/sD/fi)^(3+2*pos(sD-s14))

a13=nn13*((1-pos(sL-s13))*s13/sD/fi)^(3+2*pos(sD-s13))

a12=nn12*((1-pos(sL-s12))*s12/sD/fi)^(3+2*pos(sD-s12))

a11=nn11*((1-pos(sL-s11))*s11/sD/fi)^(3+2*pos(sD-s11))

a10=nn10*((1-pos(sL-s10))*s10/sD/fi)^(3+2*pos(sD-s10))

a9=nn9*((1-pos(sL-s9))*s9/sD/fi)^(3+2*pos(sD-s9))

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a8=nn8*((1-pos(sL-s8))*s8/sD/fi)^(3+2*pos(sD-s8))
a7=nn7*((1-pos(sL-s7))*s7/sD/fi)^(3+2*pos(sD-s7))
a6=nn6*((1-pos(sL-s6))*s6/sD/fi)^(3+2*pos(sD-s6))
a5=nn5*((1-pos(sL-s5))*s5/sD/fi)^(3+2*pos(sD-s5))
a4=nn4*((1-pos(sL-s4))*s4/sD/fi)^(3+2*pos(sD-s4))
a3=nn3*((1-pos(sL-s3))*s3/sD/fi)^(3+2*pos(sD-s3))
a2=nn2*((1-pos(sL-s2))*s2/sD/fi)^(3+2*pos(sD-s2))
a1=nn1*((1-pos(sL-s1))*s1/sD/fi)^(3+2*pos(sD-s1))
nn16=n16*un
nn15=n15*un
nn14=n14*un
nn13=n13*un
nn12=n12*un
nn11=n11*un
nn10=n10*un
nn9=n9*un
nn8=n8*un
nn7=n7*un
nn6=n6*un
nn5=n5*un
nn4=n4*un
nn3=n3*un
nn2=n2*un
nn1=n1*un
sL=sL50+sL50*us
sD=sD50+sD50*us
fi=(25/tlred)^0.2
tlred=pos(tl-25)*(tl-25)+25
```

Results:

Calculated number of repeated loading spectra in this example is b=22.344 what represents the life of 22 344 hours. This results represent the failure probability P_f=0.5. SBRA methods allow relatively precise estimation of the fatigue life for the requested failure probability. The results are summarised in the Table 2.

Table 2 Corresponding fatigue life to the probability of the failure.

***P**$_f$*	0.5	1	0.1	0.023	0.01	0.001	1.0E-04	1.0E-05	1.0E-06
b	22.34	63.15	15.43	12.48	11.41	8.98	7.77	7.70	7.69

Fig. 2 shows the reliability function RF (vertical axis) on the number of loading spectra *b* (horizontal axis) as the AntHill program output.

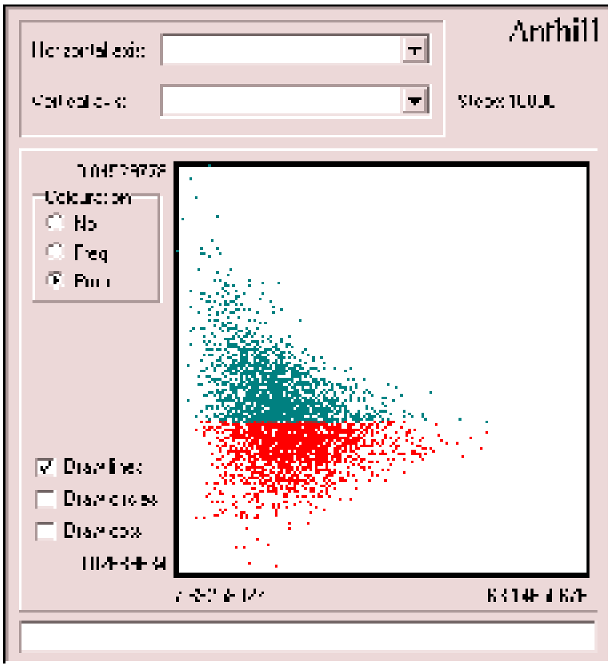

Fig.2 Corresponding reliability function RF for 1000 operating hours (see IF151801)

Comparison of Results and Conclusions:

Results mentioned above illustrate basic priority of the SBRA method. Both calculation methods give practically the same results. However the Eurocode 3 calculation can't predict the safe-life for arbitrary failure probability. On the other hand simulation by the stochastic model (SBRA) demonstrates operational time about 7 700 hours (*b*=7.7) by probability of fracture 10^{-5} what can be considered as safe enough.

References

[15.18.1] Eurocode 3: Design of steel structures, Part 1.9: Fatigue. PrEN 1993-1-9: 2002

[15.18.2] D. Radaj, C.M. Sonsino: Fatigue assessment of welded joints by local approaches. Abigton Publ., Cambridge England, 1998.

[15.18.3] Milan Růžička: Generování stochastických napětí částí konstrukce na základě měřených spekter zatížení. In: Dynamic of Machine. Coloquium Proceeding. 11-12/2/2003. IT CAS CZ Praha, 2003.

[15.18.4] Pavel Marek, Jacques Brozzetti, Milan Gustar (editors): Probabilistic Assessment of Structures using Monte Carlo Simulation, ITAM CAS CZ, Praha, 2001.

[15.18.5] Milan Gustar, Pavel Marek: *Text formatting*. TeReCo - 2nd edition.