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# Theoretical and experimental analysis of a damaging process and development of a methodology for estimating the fatigue lives of complex stressed parts

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## Characteristic of the project

Problems of fatigue damage cumulation under complex combined stress in machine parts are still less explored, in spite of a considerable progress in solution of material fatigue. The planned project sets oneself a task to find a reliable way of estimating a grow of fatigue damage in substructures exposed to combined non-proportional dynamic loading basing on detailed theoretical, numerical, and experimental analyses. A successful solution of the problem and its program implementation will offer future industrial users a mighty tool for designing and verification of new structures from the point of view of fatigue lives of complex stressed parts in service. One of the main goals of this project is a formulation and verification of a new multiaxial elasto-plastic criteria for stress-strain and damage determination on the critical plane. The criteria will be based on FEM calculations used for separate loading cases or in case of stochastic loading on analytical formula. Next fundamental step of solution is a formulation of decomposition algorithm for multi-component loading pathes and fatigue damage cumulation in metals. A comprehensiveness of the approach, tight linkage between theoretical and experimental solutions, experience and equipments of the solvers' teams make a guaranty for a high degree the proposed project fortune.

### Summary of current status

Most of machine structures and parts are subject to stresses from vibratory loading effects, which can cause their damage induced by material fatigue. The fatigue damage of material is a phenomenon, which has been known approximately since the half of 19th century. The effort to describe the fatigue damage origin and cumulation and to establish more precise and cogent models is still very actual. The cognizance that most parts are subject to complex loading for which the current computation solution (e.g. analytic procedures, reduced stressed procedures, etc.) does not have sufficient “capability to differentiate“ went hand in hand with the development of computer methods (computation using Finite Element Method - FEM) and together with the more detail description of stress and deformation states. This is how the term “multiaxial fatigue” came into being. The need of calculation steps clearly increases together with the complexity of task solution, and the use of computers is a necessary condition.

The damage induced by multiaxial stress has been monitored with increased interest approx. since the 60s of the last century. However, it should be stated that even during that 40-year period a satisfactory result has not been reached – it is clearly evident from the fact that industrial applications of newly developed methodologies are still rather rare. This conservatism results from no-confidence in new methodologies, which is connected with enormous increase of degrees of freedom, i.e. sections of solutions which using different methods of calculation can give a different result. Therefore, there is a number of criteria nowadays giving a satisfactory solution only within a certain range of solved issues. Unfortunately, there is no systematic comparison of individual deviations and methods of solution.

In the past, the Applicant's and Joint-Applicant's workplaces substantially participated in the solution of a number of problems in the area of fatigue degradation under simple and complex conditions of loading. E.g. FS ČVUT made software capable to analyse data from experiments carried out both by the Applicant and gained from literature. For the purpose of calculations, uniaxial and multiaxial methods available from the literature were programmed in order to analyse effects of various input factors. An extensive experimental program of the research of fatigue life of parts subject to combined random loading was carried out at the Applicant's workplace.

One of the basic problems, which the authors respond to in the grant application, is a formulation of a damaging parameter, i.e. a mathematical form of equation from which the resulting damage is derived, and a question what physical parameters enter it. Therefore, two issues which are rather omitted in most applications and which should be the basis of our further theoretical research, have been established as the key questions:

1. Question of local material volume plastisation in notch
2. Procedure of loading history decomposition for the entry into damaging parameter

**Ad 1**. The necessity of elasto-plastic solution is a significant obstacle for the calculation of a low-cycle fatigue where in most of a stressed part cross-section the yield strength of material is not exceeded and the material only plastize in the notch close-adjacent area. There are methods for the calculation of loading states by means of Finite Element Method (FEM), however, the calculations are carried out in an iterative manner in the entire part volume and hence substantially extend the time needed for calculation in comparison with the purely elastic solution and therefore, this solution is numerically unencompassable for the present e.g. for stochastic modes of loading. Until now, this issue has been mainly solved in such a way that loading cases when the quantity of originating elasto-plastic states is limited, are used as etalon tests, and therefore, the elasto-plastic solution of FEM can be applied. However, such assumption does not correspond to common industrial application. Nowadays already classical method according to some of analytic formulas for plastic adaptation in notch has been proposed as a practicable solution. The linear dependency between input loading and deformation or stress states is applicable here, i.e. it is possible to determine the level of local loading against external loading. Based on defined relations (Neuber method, ESED energetic method or Hardrath-Ohman method) these fictive elastic states shall be transferred to real elasto-plastic states. The problem is that for more complicated types of loading this solution does not provide a good compliance with the experiment. The key for the solution is a sufficiently functional model of multiaxial plasticity, which our research will concentrate to.

The classical theories of plasticity started to develop in the 40s and 50s. They can be divided into several groups. There are classical models utilizing just the plastic potential, yield surfaces, isotropic and kinematic strengthening of material. The development has passed through multi-surface models to currently preferred double-surface and single-surface models with non-linear strengthening on the basis of Armstrong and Frederick model [1]. Further more precise specification now relates first of all to searching for a suitable criterion for the evolution of kinematic strengthening where the development is directed to the superposition of several such models in order to involve a wide area of considered deformation range. In the 70s there were so-called endochronous theories of plasticity that appeared under the leadership of Valanis, see e.g. [2]. They assume that actual status of stress or deformation in material is the functional of the entire previous deformation history. In the basic version, the solution independence on the existence of the yield surface was strongly emphasized. However, it started to appear that this theory has a lot in common with the classical solution and recently, both solutions have reached a very similar result. The third method of solution mentioned herein includes criteria resulting from the Gurson definition of plastic potential [3]. It is based on the determination that a porous structure is formed in material at extreme pressures (forming etc.) and the material becomes sensitive to hydrostatic pressure. Currently, these so called "cavity" criteria are subject to extensive development, however, they are rather applied to irreversible loading paths.

At present, there are different approaches as regards the solution of plastic adaptation in notches. One of the ways is to apply the Prandtl-Reuss equations (constitutive description of material elasto-plastic response) for the calculation of stress or deformation components while considering some generalized form of Neuber or another rule as an applicable form for equivalent stresses [4]. The insufficient number of relations for the number of unknowns represent a complication here (also with the plane state of stress on the part surface there is still one equation missing. This basic scheme of solution then breaks down in dependence on various authors´ assumptions on the quantity of proposed criteria (e.g. [5], [6]) where the definition of this missing equation is completed in various ways. Another interesting way is to use description of boundary surfaces, which corresponds to models used for the calculation of elasto-plastic response in FEM, only with the difference that there is an effort to eliminate this iterative (a thus time-demanding) way of calculation. A typical example can be represented by [7]. Thus it is ensured the fatigue calculation need not be directly integrated into the FEM analysis and at the same time it is possible to record continuous changes in material behaviour during the part life time. The datum only consists in the above-mentioned elastic FEM calculation. Therefore, we wish to concentrate out attention to the formulation and verification of the new procedure.

**Ad 2** The other point of the theoretical solution the grant application focuses on relates to the method of loading decomposition [13]. So far an effort is evident also in the literature to come out of the conditions, which proved to be applicable for uniaxial loading, in particular from the rain-flow method. Description of stress or deformation state on a particular plane is often used for the calculation of damage. One of the methods includes decomposition on a normal or shear component of loading state and the remaining stress component is then allocated to the just decomposed oscillation [8]. Another possibility is to perform the same on one of equivalent values of stress or deformation [9]. The advantage here is that the decomposition result does not change even for various displacements of examined plane. The solution presented by Brown and Wang [10] performing the decomposition based on a relative deviation of equivalent deformation has a similar character. The case when a shear component is to be decomposed is substantially complicated when during more complicated loadings the end point of the shear vector copies a complicated trajectory. Methodologies have been established how to analyse this fact [11], however, the problem is that the authors again come out from limiting assumptions and there is no definition of when the end of loop is reached. There is usually an unrealistic assumption that the scribed trajectory always represents the end of loop. However, our own experience has shown that this algorithm [11] is rather inclinable to numerical errors and is rather long. Therefore, it is really needed to revise these procedures**.**

## Project goals

The general goal of the submitted grant project is to increase reliability of fatigue life prediction of complex-loaded machine parts with the use of computer techniques and large material databases.
The goal of the submitted project numerical part is to design and verify new satisfactory algorithms and methods for the solution of the two above described issues and implementation thereof into the current program [12], [13], [14], [15], [16] so that it is practically possible to solve needed tasks of complex-loaded parts and to compare individual methodologies with the experiment. In particular, it means to formulate and verify the new "multiaxial elastoplastic criterion” or a procedure for the determination of data on the critical plane based on the files of FEM results of individual loading states obtained by linear (elastic) calculations. Another concretized goal, based on the test results obtained on partial loading paths and with generally complex loading, is to formulate the procedure of “decomposition” or an algorithm of damage evaluation description on a general loading way which would allow to make calculations of complex-stressed components life.

The goal of the research experimental part is to complement the existing pieces of knowledge and information about fatigue behaviour of steel specimens for harmonic loading paths (combination of tension-pressure-torsion during harmonic loading without and with phase shift) by other specially designed periodic loading paths (see part c) which would be the basis for the above computer analysis. Another goal is to investigate the course of deterioration and to obtain experimental results for the complex loading path, which is to be the subject of computer decomposition. The test results will also include information about the position of place of crack origin (critical point) and about the speed of defect propagation, which will also be subject to theoretical evaluation. Another goal includes cognizance of the loading processes statistical properties effect on the fatigue life.

**Conceptual and methodical approaches**

As already stated, the actual structures are in operation subject to time-variable force effects, which after long-term action can cause material fatigue. A fatigue microcrack arises in a critical place of the structure and its further propagation results in general deterioration of the supporting cross-section. The main factors effecting the fatigue process include namely:

- part shape (its geometry and size)
- loading method (from the viewpoint of arisen stresses type, their time course)
- type of used material (tough, brittle)
- technological processing (surface treatment, surface layer heat treatment, residual stresses)
- effect of environment (effect of temperature, corrosion atmosphere, etc.)

The research is aimed at the quantification of namely first three effects, the technological aspect and working environment assume standard conditions. Various methods can be used to prove sufficient fatigue resistance, which can be mutually combined, for the optimization of costs on one hand, and also for mutual support and feedback of computation and experimental methods on the other hand. These approaches include namely:

- nominal approach
- local approaches through elastic or actual stresses and deformations in notches (for the phase until fatigue cracks initiation)
- fracture mechanics approach (for the phase of crack propagation)
- experimental proofs

The advantage of classical procedures based on nominal approach is in their simplicity (resulting from analytic relations of elasticity and strength ), however, they require experimental determination of fatigue curves of real parts, or at least model specimens, for every type of loading.

Local methods are preferred namely with the possibilities to use computer techniques and Finite Element Method (FEM) application for the calculation of stresses and deformations (both for linear and non-linear calculations). The grant application is aimed particularly at further development and verification of local methods, namely for multicomponent loading.

According to present knowledge [13], the influence of a loading path on the fatigue damage progress is very important. The *loading path* (often *deformation path*) means a characteristic curve describing the relation between relative strain tensor components (event. stress tensor) on a critical plane during loading (time) (see fig. 1).

The cause of the loading path effect on the damage is that during non-proportional loading due to rotation of main directions the deformation strengthening occurs higher that during proportional loading. In addition to that the analysis shows [10] that with the same amplitudes of loading it applies that the larger surface is scribed by the loading path, the higher the material strengthening is.

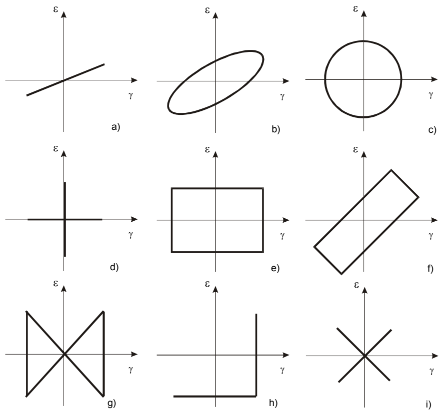

Fig. 1 Various types of periodic loading paths

In principle, these loading paths can be divided into periodical and non-periodical. In order to quantify (event. separate) the damaging effects from individual types of relative deformations, it is necessary to investigate the damaging mechanisms at different loading paths.

For our own analysis of the above described effect, we will use the results obtained in the past for the most often used proportional loading (harmonic in-phase tensile and torsion stress, see Fig. 1a) as well as periodic non-proportional loading with general phase shift, or a 90° shift (see Fig. 1b and 1c), which were one of results of the grant 101/99/0103 solved in the years 1999-2000 [20]. One of partial goals of the proposed solution is to complete these results by other loading paths, the example of which is shown in Figures 1d) to 1i). Their final specification will be specified after performing computer simulations and after testing the possibilities of experimental techniques. By completing these experimental bases for the same type of tested material and test specimen (i.e. for steel pipe specimens of diameter 30 mm and wall thickness 2 mm, material 11 523 with a hole of 3 mm in diameter), we will obtain bases for more complex view at the damaging mechanism on the critical plane and a possibility to design algorithms for the multiaxial method for inclusion of closed loops deterioration effect (an analogy to unidimensional rain-flow method).

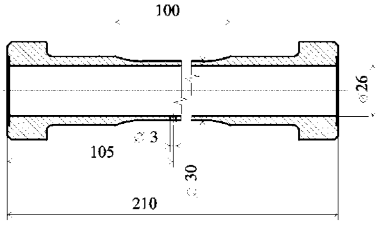

Fig. 2.: Test specimen for combined stress

As the general courses of loading are usually stochastic, this method should be generalized for non-periodical processes. As the needed number of tests would grow into innumerable set, some published experimental results will be used for the analysis, which after the analysis will be transferred to used material and verified by one type of test for more complicated loading path. In actual case the loading path will include several combinations of closed „nested“ multiaxial“ loops so that numerical simulation by elastoplastic calculation using FEM could be performed.

The local elastoplastic response in notch will be simulated using Finite Element Method (FEM) also for the above-mentioned partial loading paths. Elastoplastic analysis modules in ABAQUS programs will be used for the calculation. Results also obtained by means of the PMD program (developed by ÚT AVČR) will be compared for the selected case.

The synthesis of partial result files for individual loading states is one of other issues when FEM modelling and when predicting deterioration and fatigue damage of real parts. The problem consists in the fact that complex operation forces acting on a given part are considered separately in the FEM calculation models (they are usually substituted by a „unit“ effect). Subsequently, the resulting stress state for actual operational loading is composed from these calculated partial loading states. Considering linear transfer matrix for individual force effects and for linear behaviour of material the resulting state of stress can be gained very simply by linear combination of calculated partial loading states. Then e.g. for equivalent stress in place *s* of the structure the following relation will apply: $\sigma_e = \sum_i \sigma_{e,i}(t,s) = \sum_i F_i(t) \cdot c_i(s)$. At proportional loading when it is possible to determine the rates between loading components $k_i = F_i / F_1$, it is possible to determine unambiguous coefficients of loading transfer into the critical place $c_e(s) = \sum_i c_i k_i$, so that $\sigma_e(t,s) = F_1(t) \cdot c_e(s)$.

For local approaches and for the deformation response analysis with the assumption of non-linear behaviour in the area of notches this procedure cannot be used and it is necessary to search for non-linear transformations to critical node or to use a linear formula and to apply some of multiaxial models of cyclic plasticity to the resulting tensor. As even the FEM calculations cannot be performed for capacity and time demands due to a large quantity of oscillations (e.g. at general stochastic process), it is necessary to formulate a „multiaxial Neuber rule", as the analogy to the rule used for uniaxial procedures could be called. Whereas these methods are routinely applied for the uniaxial approach, see e.g. [17] or a summary in [13], there is no procedure for multiaxial stress as already mentioned in paragraph ***a***. The grant intention is to solve this question by arithmetic simulation and experimental verification on tube specimens.

Global and local stresses in the notch area will be analysed by means of FEM calculations and the critical place and its critical plane for individual loading states will be searched for. From them, a theory for the evaluation of damage for resulting loading states arising in each step of the loading effects time course will be searched. The critical place will be predicted by calculation as well as the part life time both for individual partial loading paths and at complex loading cycle. The selected loading cycle shall be sufficiently representative to allow verifying the activity of the above described multiaxial rain-flow method and moreover, be manageable by the FEM computer simulation.

The theoretical procedure will be based on the usage of description of boundary surfaces of plasticity and interaction of isotropic and cyclic strengthening at multi-component states providing homogeneous isotropic plastic behaviour of material (i.e. structural steels). The theoretical analysis of the problem was indicated for the approaches of various theories in [18].

Experimental tests will be provided by ÚT AVČR in the Material Diagnostics Centre in Plzeň and partly by external partner SVÚM Praha.

### Expected result and its utilization

The project proposal results from a deep analysis of practical needs. The project organically connects theory with experiment thus allowing to detect basic effects contributing to the damaging mechanism under combined stresses. It is assumed that the project solution will lead to the increase of current level of knowledge, namely in the following areas:

- extension and improvement of knowledge concerning the development and acting of fatigue damage mechanisms on machine parts when exposed to combined proportional and nonproportional random loading,
- development of a new method of analysis of time course of stress-deformation response on critical plane which would consider the multiaxial state of stress and actual history of loading
- development of a new method and algorithm to take into consideration the plastic adaptation in notches and thus the possibility to develop methods of local approach for the prediction of machine parts life time at proportional and nonproportional random loading
- development and creation of new program modules and integration thereof into the existing program MAXA for uniaxial and multiaxial prediction of machine parts life time.
- preparation of improved methodology for fatigue life tests under combined random stress

The project solution results can be utilized in various industrial branches, namely in transport technology and also in other branches utilizing the dimensioning of structures subject to dynamic loading processes and when predicting fatigue life. The solution results can also be utilized within education at technical universities (ČVUT and ZČU) and training of new doctors.

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