NEW CONCEPT OF A LOCAL ELASTIC STRESS APPROACH FOR FATIGUE LIFE CALCULATION

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Abstract: Principles of different approaches to fatigue life prediction until the crack initiation on aerial constructions are described in this report. Local approaches to lifetime evaluation are analysed on the basis of stress and strains results of a FE-analysis. The nominal approach, which enables to use the usual stress fatigue curves, is modified for usage in local approaches. The proposed LESA method makes use of stress fields in FE-results for determination of fatigue damage and its further visualisation on FE-models.

1. INTRODUCTION

The calculations utilizing the finite element method (FEM) are among integral systems of computer aided design, calculation and manufacturing, which serve for design of airframe components. The FE-analysis constitutes a base for any next check of limit states including fatigue life prediction and determination of resistance to the crack growth. Although the research of mechanisms and processes of damaging manifested substantial progression in passed decade, great hopes in creation of general damaging model, which would be applicable to numerous and various service conditions, to a complicated geometry and to different materials of engineering products, are not appropriate. The classic approaches based on the evaluation of nominal stresses at critical areas of constructions and on traditional stress fatigue curves are still often used for prediction. Moreover, methods of a local approach, which take the result file of FE-analysis as a direct data source, are utilized. Such solution requires much more precise theoretical (experimentally tested) model of damaging, that would respect both the phenomena of multiaxial stress states, which arise in local critical places, and of multiaxial loading history. Lack of experimental data in the case of multi-loading states, which demand tests of unaccustomed types, restrains among others a faster expansion of such approaches into the engineering practice. Nevertheless, there are vast experimental experiences and databases, which were obtained throughout the past years, on the other hand. They contain numerous S-N curves of classic and even wholly specific construction variants and solutions. Thus, missing their presence could be unwise. Perfection of engineering techniques and calculation methods, which can utilize such data, is still needed – above all in the case of primary design of constructions. One has to be aware of its accuracy naturally and bear in mind critical judgment to these methods of fatigue strength and fatigue life estimations. Experimental verification of the lifetime of typical construction elements and assemblies and of prototypes stays to be the integral part of most projects. The experimental solution can lead to substantial improvement of computational estimates and to more precise evaluation of service reliability.

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2. BASIC APPROACHES TO FATIGUE LIFE PREDICTION

2.1. Prediction by nominal stress analysis (NSA)

The NSA (Nominal Stress Analysis) method is the historically oldest way of fatigue life design. It has been developed together with the evaluation of so-called notch strength of components, which was based on relation of stress extremes in notches to levels of nominal straining (available from analytic calculations). The extensive database on stress states in notches and on fatigue endurance of notched components was created throughout several decades and now is being utilized in engineering industry and in standards as well. These valuable data can be taken as a base even in the case of more modern types of solution or within utilization of results of FE-calculation. A graphical scheme of the design process is depicted in Fig. 1.

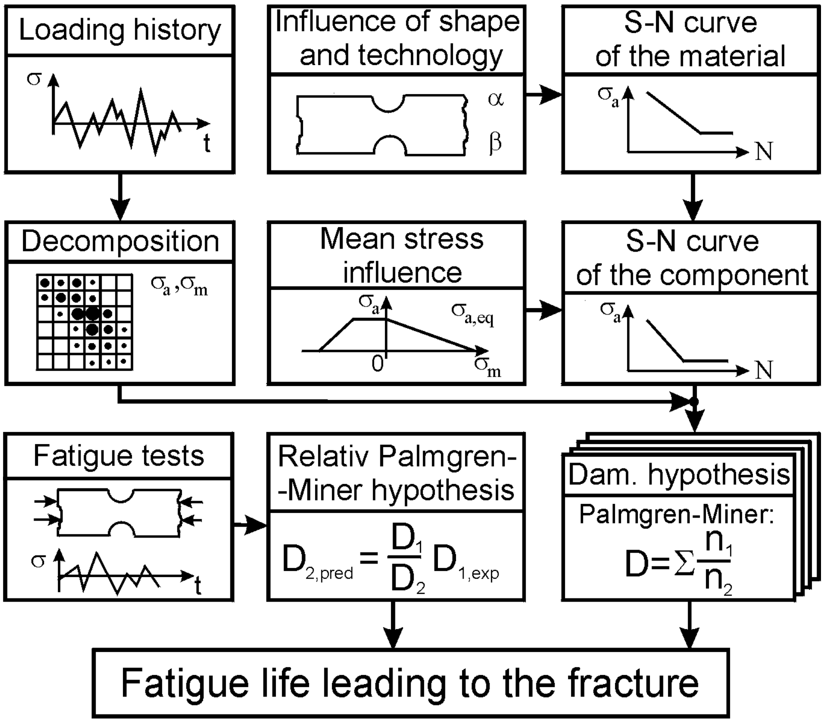

Fig. 1 Flow diagram of NSA (Nominal Stress Analysis) design method.

Here can be seen, that necessary input information is:

- History of nominal stresses in some critical place of the component throughout the time. The history is processed by a method of decomposition (e.g. the possible output can be the rain-flow matrix in the two-parametric form). The prediction results can be influence substantially by the type of decomposition. The rain-flow method is recommended as the most realistic one. The work with effective stresses (according to some of strength theories) is the most often used method in the case of prediction within multiaxial loading.
- *Stress fatigue curve* (Wöhler curve, S-N curve) valid for critical area of designed component is obtained or deduced from experiments. Due to lack of data, it is often derived from S-N curves for notched or even smooth specimens – so the resulting data can be called as *synthetic fatigue curves*. Many factors concerning portability of model tests to some real component have to be reconsidered in such cases.

- Haigh's or Smith's diagram for mean stress correction of fatigue curves or some relation for the computation of the amplitude of equivalent stress.
- Correction factors have to be considered for inclusion of other effects known from experimental databases of materials - the influence of stress concentration and gradient, of absolute size of the component, of surface quality, of residual stresses etc.

Lifetime prediction itself is based on the fatigue damage accumulation according to some of the hypotheses. Basically the Palmgren – Miner hypothesis is used. Resulting lifetime, which is predicted by the NSA method, is treated as the mean lifetime (its median) with probability of fracture occurrence P = 50%, where the fracture is thought to be complete in the critical area. This presumption stems from the ideas, that tests of notched specimens are made until the final rupture and that the period of crack growth is relatively shorter in contrast to the initiation phase.

2.2. Prediction by local elastic-plastic strains and stresses (LPSA)

The basic idea of computational lifetime estimate of real component with notches is based on the following consideration. The component is exposed to a harmonic loading with the stress and strain amplitudes in referential (nominal) area σ_{an}, ε_{an}. The failure can be expected in the critical place (notch), where the local stress and strain amplitudes $\sigma_{a\nu}$, $\varepsilon_{a\nu}$ are. Since the method is based on deformation behavior in a volume of material, which is bordered (and loaded) by surrounding elastic matrix, strain fatigue curves (Manson – Coffin curves) for the strain-controlled loading are used. The failure will arise after $2N$ half-cycles, which lead to a point on a Manson – Coffin fatigue curve of utilized material (the curve is obtained on a smooth specimen under symmetric cycling) and to a loading by parameters $\sigma_{a\nu}$, $\varepsilon_{a\nu}$ in the examined critical place of the component.

The LPSA (Local Plastic Strain and Stress Analysis) method is schematically shown in Fig. 2. The input information is:

- The loading history in the form of sequence of individual extremes of loading process throughout time, or an already processed record of the one- or two-parametric loading spectra (rain-flow matrix) can be available. The rain-flow method is often utilized for the signal processing as before.
- Fatigue properties and parameters of used material are represented by the strain fatigue curve (Manson – Coffin fatigue curve). The curve corresponding to symmetric cycling is expected.
- The constitutive relation between stress and strain values under cyclic loading (*cyclic strain curve*).

Usually the elastic-plastic solution in the local volume of material (notch root) is not available for every loading extreme, since it would imply a solution of an elastic-plastic stress analysis throughout the real time under complex loading. Thus, approximate conversion relations were deduced. The Neuber's rule (or its generalization [1]) is most often used for such conversion or the equivalent energy method can be used [2].

Since the experimental results show, that asymmetry in load cycle (with the coefficient of asymmetry R > -1) reduces lifetime in contrast to symmetric cycling, it is necessary to take the mean stress correction into consideration. There are many diverse approaches in various methods, how to do that [3].

The lifetime prediction is also based on the type of fatigue damage cumulation, which usually has a linear type. If one disposes with experimental tests on a prototype or on some similar component, the relative Palmgren – Miner rule can be taken. The determined mean lifetime is interpreted as the lifetime until the initiation of a crack of a technical size.

To get the lifetime until the final breakage, the number of cycles necessary for crack propagation has to be added. It can be obtained by methods of fracture mechanics.

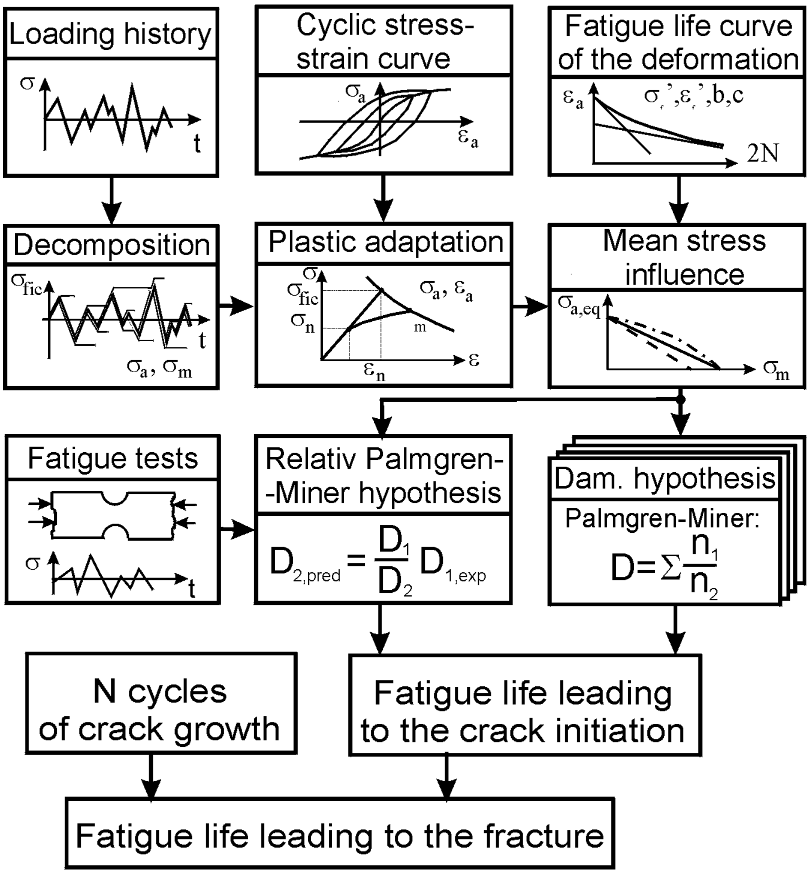

Fig. 2 Flow chart of LPSA (Local Plastic Strain Analysis) method utilization in design process.

The advantage of LPSA is the possibility of a direct utilization of FE-results in the form of local values of stresses and strains in examined critical areas. Nevertheless, the knowledge of Manson – Coffin fatigue curves is necessary (from strain-driven tests) for any calculation of fatigue damage. The database of such tests has substantially shorter historical tradition compared to the stress-controlled experiments (S-N curves). The LESA (Local Elastic Stress Method) was elaborated for the utilization of classic S-N curves for local approach methods.

3. METHOD OF LOCAL ELASTIC STRESSES

The absence of experimental data in primary phases of design work, the possibility to use the correlation between static and cyclic characteristics of materials and the application of numerical methods of stress analysis (mostly FEM) at last formed the first computational procedures in 70s, which stemmed from stress analysis in exposed local areas of constructions – mainly in notches. It is possible to work with fictitious elastic stress tensor components or to determine the real elastic-plastic state in such localities. The LESA (Local Elastic Stress Method) method is built on Hooke's elastic stresses in places of stress concentrators. It de facto modifies the NSA method. The original method according to Crews and Hardrath [4] lead to strongly conservative results. We adapted the approach for purposes of damage prediction in localities with different stress extremes and gradients, so that it is now equivalent to the NSA method and can utilize the same basic fatigue curves. Whereas the NSA method was related to nominal stresses and fatigue curve was accommodated to some particular concentrator (leading to a shift of S-N curve towards the lower values of stresses), the LESA method is processing only one S-N fatigue curve corresponding to the smooth specimen. By contrast, the local elastic extremes of stresses are accommodated so that they respect the real notch factor of critical place. The fatigue damage computation is performed using any method of damage cumulation. The damage corresponds to a lifetime until the creation of technical macro-crack in the checked area. The macro-crack term denotes the crack, which is greater than the elements of material structure (the size of grains) and by which the further growth can be described with the use of fracture mechanics. Mostly such size is greater than 0,5-1 mm. The computational flow chart of described lifetime prediction is depicted in Fig. 3.

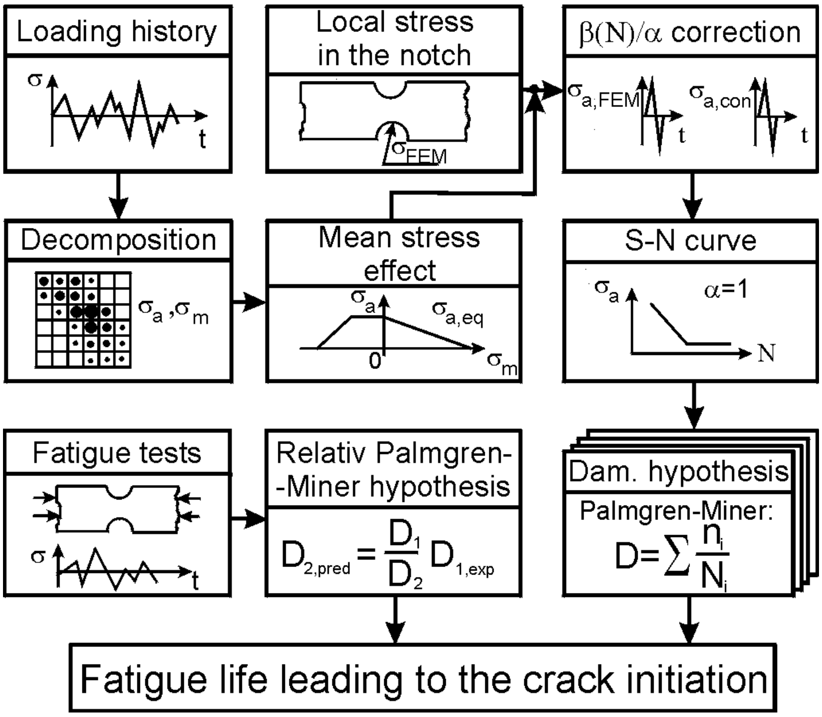

Fig. 3 Flow chart of LESA (Local Elastic Stress Analysis) method.

3.1. Base of LESA method

According to a theory of a weak link (the Weibull's probabilistic theory of serial linked elements) and to criteria of stress states similarity of two parts, the fatigue limit of an examined notch can be set in relation to the size of an *exposed volume*. It demarks a volume of material in the vicinity of the concentrator, where the *function of equivalent stresses* exceeds some definite limit fatigue value of the stress amplitude. Values of relative stress gradients and of exposed volumes are tabulated for many standardized notches. Because the search for the size of the critical exposed volume can be in common practice very troublesome and since the stress integration over such volume has not been easily done yet, the following method was chosen.

The course of an *evaluative stress* is determined in the examined area on some suitable cross-section. The value of the greatest principal stress σ_{max} or of the maximum shear stress τ_{max} is utilized as the evaluative stress according to the particular hypothesis of

equivalent stresses. Both the extreme and nominal (mean) values of stresses in the notch are specified from the course of stresses. That way the shape factor α in the particular section can be defined. As next a gradient of a stress decrease in the root of the concentrator is evaluated. A number of numerical influences can be discussed in process of practical evaluation of stress gradients. Thus, e.g. the dependency of results of concrete stress tensors and of related quantities in the notch root obtained from the FE-calculation, on a density of the mesh and on types of used elements (the number of integration points, the extrapolation of the stress tensor from integration points to the nodes and to the borders of element) can be investigated. Determination of stress gradients is generally done by a calculation over several elements (at least two of them) in the direction of its lowest value. The relation of stress gradient to the extreme stress value in the examined section leads to the *relative stress gradient* γ [1/mm].
The rule according to Siebel and Stiller acquitted well for notch factor determination. The Siebel – Stiller's curves are approximated for any effectual algorithmization by the following relation [5]

$$\frac{\beta}{\alpha} = 1 + \sqrt{\gamma} \cdot 10^{-\left(0.35 + \frac{R_e}{810}\right)}, \tag{1}$$

where R_e corresponds to the yield stress of material in MPa.
If the notch factor is already known, the shift of the fatigue limit for smooth specimen σ_C can be done in a ratio

$$\beta = \frac{\sigma_C}{\sigma_C^{\times}} \tag{2}$$

and the fatigue limit of notched bar $\sigma_C^{\times}$ can be thus estimated. To restore whole fatigue curve of a notched specimen, the generalized notch factor for a general number of cycles N was defined as

$$\beta_N = \frac{\sigma_A(N)}{\sigma_A^{\times}(N)}. \tag{3}$$

A dependency of the notch factor on the number of cycles leading to the damage was chosen according to Heywood [6]

$$\beta_N = 1 + (\beta - 1) \cdot \mu(N), \tag{4}$$

where sensitivity to the number of cycles is set as

$$\mu(N) = \frac{(\log N)^4}{B + (\log N)^4}. \tag{5}$$

If the β_N factor is known, the utilization of eq. (3) returns the whole fatigue curve and thus allows the successive fatigue damage calculation according to the NSA method.
The FE-calculation of stresses in the construction allows obtaining of not only the integral information on nominal stresses in the examined section, but also the possibility of determination of local extremes of the stress tensor and of its variations in a closest area around the concentrator (i.e. the possibility to set the stress gradient). Since we expect a derivation of the stress state from a linear combination of particular loading states in any complex construction (each loading state is computed from a FE-model loaded by a unit load), we reason to utilize only elastic behavior of material and the elastic FE-solution.

The here presented LESA method is analogous to the NSA method. The difference is, that the fatigue damage calculation is not based on the nominal stress on the component, but on local elastic stresses in the notch, which were obtained from FEM. Some fictitious fatigue curve is assigned to each examined place. Such curve is derived from a basic S-N fatigue curve valid for the smooth test specimen. The process of its derivation respects the concrete size of concentration, stress gradient and variable notch sensitivity of material β_N (dependent on the number of cycles) as well. The situation is depicted in Fig. 4.

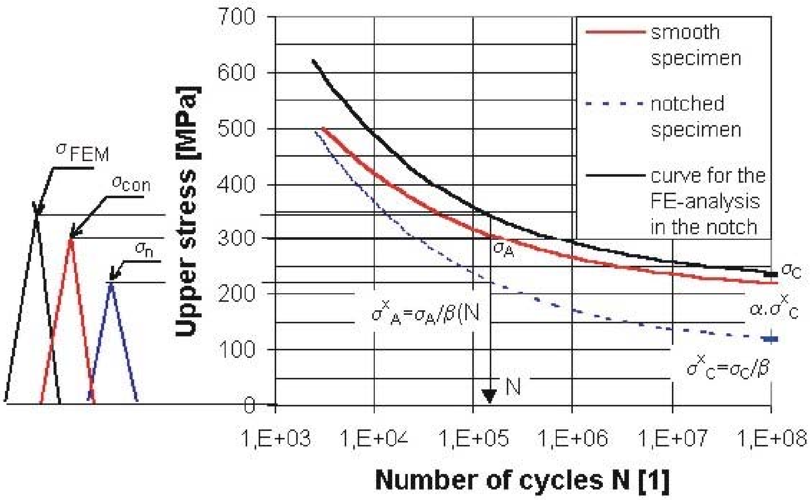

Fig. 4 The principle of local stress correction in the notch from the S-N curve of the smooth specimen.

Since not nominal stresses σ_n, but local elastic stresses σ_{FEM}, are acquired from the FE-calculation, the construction of a fictitious fatigue curve for every local concentrator has to be done. Usually only the S-N curve for the smooth specimen (with fatigue limit σ_C) is known. This curve is corrected for a given notch factor towards lower values of stresses (i.e. similarly as the S-N curve with fatigue limit σ_C^x in the NSA method). Then a shift follows towards higher values by the ratio of shape factor α. This approach can be simplified, so that a work with only one S-N curve valid for the smooth specimen is needed and the shift is only fictitious. The solution of such simplification is acquired by a correction of extremes of elastic stresses σ_{FEM} from to σ_{con}. It can be seen in Fig. 1, that if the same lifetime (i.e. number of cycles N) has to be calculated with the utilization of all curves, the S-N curve of the smooth specimen must be shifted upwards in the ratio of factors α / β_N. Because the S-N curve of the notched specimen can be derived with the use of generalized notch factor β_N for a given number of cycles, the relation is valid for the stress value in the notch, which was calculated by FE-method.

$$\sigma_{FEM} = \alpha \cdot \sigma_n = \frac{\sigma_{con} \cdot \alpha}{\beta_N}. \tag{6}$$

The size of corrected stress can be thus expressed and we obtain the equation, according to which the stresses from FE-results must be converted

$$\sigma_{con} = \sigma_{FEM} \cdot \frac{\beta_N}{\alpha}. \tag{7}$$

Corrected values of stress amplitudes from FE-results will be applied for the damage calculation. Their application will be based on the utilization of the S-N curve for the smooth specimen.

3.2. Synthetic fatigue curves

The curves of used material (e.g. the ASTM 300M steel according to MIL-HDBK catalogue [7] here) are taken as a base for derivation of fatigue curves valid in critical places of constructions. The mathematical description of fatigue curves and of their parameters for smooth and specific notched specimens (i.e. with a variable shape factor α) are presented in Tab. 1.

	Shape factor α	Fatigue curve description $N \cdot (S_{eq} - S_C)^w = C$	Exponent n for equivalent stress computation $S_{eq} = S_{\max}(1-R)^n$
300M steel	*1.0*	*logN = 10,58 – 3,02 · log(S* $\log N = 10{,}58 - 3{,}02 \cdot \log(S_{eq} - 75{,}0)$	*0,39*
	2.0	$\log N = 12{,}87 - 5{,}08 \cdot \log(S_{eq} - 55{,}0)$	*0,36*
	3.0	$\log N = 9{,}52 - 3{,}00 \cdot \log(S_{eq} - 25{,}0)$	*0,50*
	5.0	$\log N = 9{,}61 - 3{,}04 \cdot \log(S_{eq} - 15{,}0)$	*0,52*
	N ... lifetime in number of cycles S_{eq} *... upper stress of equivalent cycle [ksi]* S_{max} *... upper stress of loading cycle [ksi]* S_c *... fatigue limit [ksi]*		*n ... exponent for equivalent stress computation* *C, w ... fatigue curve parameters* *R ... coefficient of cycle asymmetry* S_{min} / S_{max} ***Remark**: 1 ksi = 6.894 MPa*

Tab. 1 The description of fatigue curves with various values of shape factors (300M steel, from [7]).

The analytic equations for a factor β_N description were further utilized for derivation of *synthetic fatigue curves*. These curves enable to derive any required fatigue curve for a critical area (described by a concentrator α and a stress gradient γ) from one fatigue curve of the smooth specimen. More fatigue curves related to various concentration parameters establish *family of fatigue curves*. Here also the following usage of a single base fatigue curve for derivation of fatigue curve at concrete local place in the construction is presented.

The experimentally obtained fatigue curves of ASTM 300M steel are acquired from MIL-HDBK-5D catalogue [7] (valid for a symmetric cycle in tension-compression for various shape factors α). Synthetic fatigue curves of the specimen with typical stress concentrators were derived from the base curve. The model notch was presented by a specimen with a circular neck, where the relation $\gamma = 2 / \rho$ is valid, being ρ the symbol of the notch radius. The Siebel-Stiller law, given previously in (1), was generalized to formalize the relation between influence of effective notch factor (presented by the β factor) and shape factor α

$$\frac{\beta}{\alpha} = 1 + \sqrt{\gamma} \cdot 10^{-(K1)} \tag{8}$$

Also the dependency of notch factor on the number of cycles to the fracture was generalized

$$\beta_N = 1 + (\beta - 1) \cdot \mu(N)$$
$$\mu(N) = \frac{(\log N)^E}{B + (\log N)^E} \; . \tag{9}$$

It was proved, that for the satisfactory precise and general approximation of fatigue curves, the number of parameters must increase, above all to reach better fitting of $\mu(N)$ function on the relative gradient γ. The second part of eq. (9) was thus proposed with following parameters

$$E = 4 \cdot \gamma^{K3}, \quad (10)$$

$$B = \left[\frac{1}{(1+\gamma)^{K4}} \cdot \frac{K5}{R_m} \right]^2. \quad (11)$$

Fig. 5 depicts the comparison of commented catalogue curves and the derived ones. The goal is to obtain the smallest possible difference in the area of middle and higher notch factor and in the range of cycles between 10^4 and 10^7 cycles. It can be clearly seen that the coincidence is satisfactory - especially towards the original sources, where the significant scatter of experimental points is clear, and to the fact, that the punctuality of result approximation by a mean curve is affected with certain error. The parameters of a studied approximation are given in Tab. 2.
With the utilization of proposed parameters, the fatigue curves family of circular specimens with various factors α was computed (Fig. 6). To describe the effect of number of cycles (i.e. the $\mu(N)$ factor) on the size of different notch coefficient β, the β_N dependencies were depicted (Fig. 7). It can be traced that the notch factor progressively decreases with higher concentrations on the specimen for lower numbers of cycles.

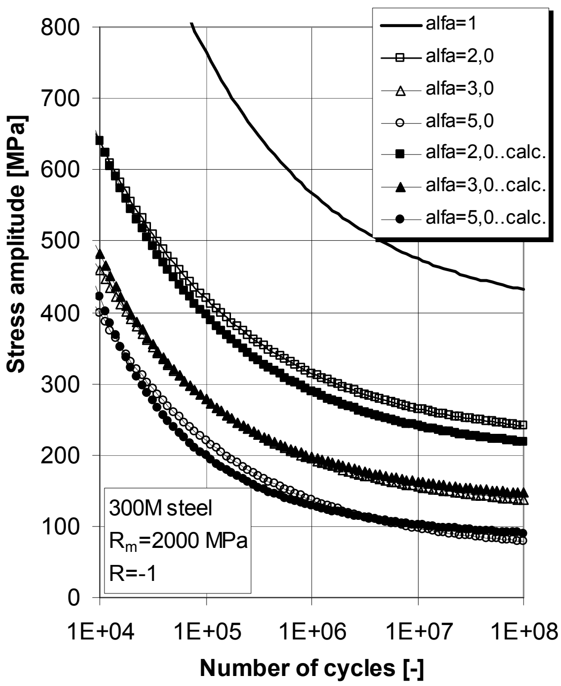

Fig. 5 Derivation and comparison of fatigue curves of notched specimens (valid for 300M steel).

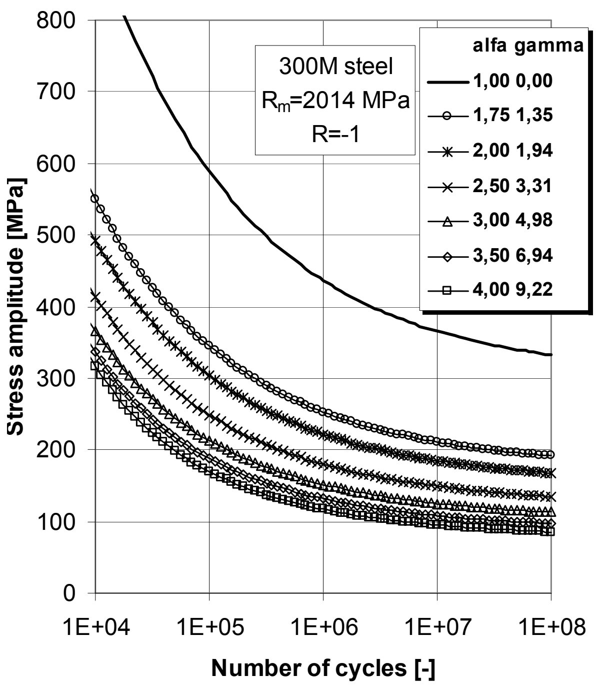

Fig. 6 Fatigue curves family (300M steel).

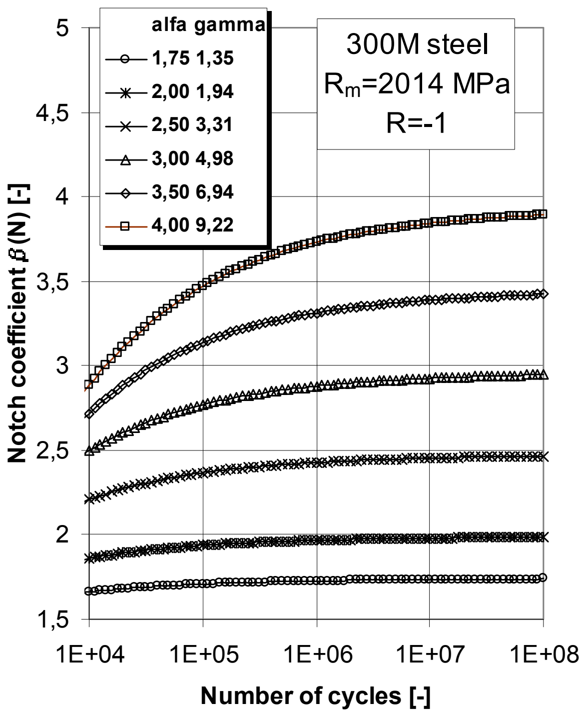

Fig. 7 The dependency of notch factor β_N on the number of cycles N (300M steel).

Parameter	*Value*
K1	2.4093
K3	0.1
K4	-0.98
K5	6000

Tab. 2 Parameters used for derivation of fatigue curves (300M steel).

3.3. Mean stress effect

The following relation representing the effect of cycle asymmetry is used in MIL handbooks [7]

$$\sigma_a^n \cdot \sigma_h^{1-n} = konst\,. \tag{12}$$

Having the fatigue test with the pulsating load cycle ($\sigma_a = \sigma_m$, $R = 0$) as the referential test, we label the equivalent amplitude $\sigma_{a,eq}$ and the following relation is valid for a general cycle

$$2^{1-n} \cdot \sigma_{a,eq} = \sigma_a^n \cdot \left[\sigma_a\left(1+\frac{1+R}{1-R}\right)\right]^{1-n} = \sigma_a\left(\frac{2}{1-R}\right)^{1-n}. \tag{13}$$

The formulation for setting of the amplitude of equivalent pulsating load cycle related to a general cycle can be hence derived

$$\sigma_{a,eq} = \sigma_a \cdot \frac{1}{(1-R)^{1-n}}\,. \tag{14}$$

If the value of upper stress of the equivalent pulsating cycle $\sigma_{h,eq}$ is desired (e.g. in the aircraft construction), the equation (14) must be further arranged

$$\sigma_{h,eq} = 2\cdot\sigma_{a,eq} = 2\cdot\sigma_a\cdot\frac{1}{(1-R)^{1-n}} = 2\cdot\sigma_h\left(\frac{1-R}{2}\right)\frac{1}{(1-R)^{1-n}} = \sigma_h(1-R)^n\,. \tag{15}$$

This equation can be found among methods of the fatigue curves analysis in the MIL standards [7]. The well known relation according to Oding can be obtained from this solution by setting the exponent n value to $n = 0.5$

$$\sigma_{h,eq} = \sqrt{2 \cdot \sigma_a \cdot \sigma_h} \quad , \tag{16}$$

If the value of exponent *n* is evaluated for different asymmetry coefficients *R* from experimentally obtained fatigue curves, it may vary. Thus e.g. by 300M steel its value moves between 0.36 and 0.52. Such reality markedly complicates any work with the family of synthetic fatigue curves, because here should be done an interpolation of exponent *n* values for any general curve from family. The mathematical model, which would interpolate in all variables α, γ, *n* is still in a phase of development.

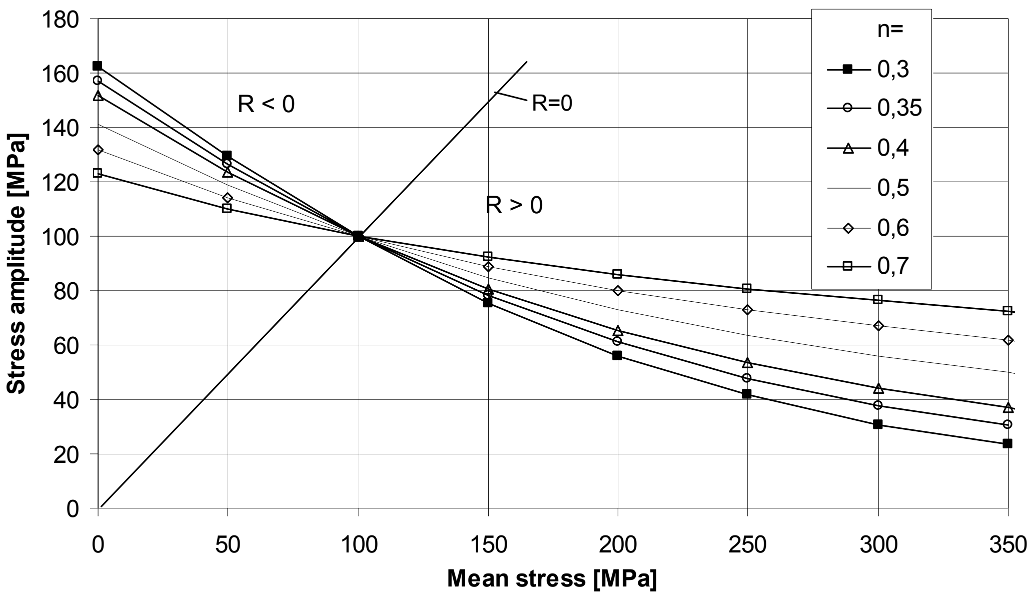

Fig. 8 Effect of the exponent *n* size on the course of a mean stress correction curve.

In the current phase of calculations, such value of exponent *n* is chosen, so that the equivalent cycles cannot be substantially underestimated. The comparative Haigh's diagram Fig. 8 was created therefore. The figure shows the size of a general cycle (depicted as a point lying on chosen curve, e.g. A[σ_a, σ_m]), so that the equivalent pulsating cycle is precisely 100 MPa. Obviously, if the equivalent cycle is set according to the Oding's formula (16) (n = 0.5), then the non-conservative equivalent values will be acquired for cycles with the coefficient R < 0, if the fatigue curves lead to n > 0.5. In contrast, the comparison with values valid for n = 0.5 gives difference up to 9% for the value of n = 0.68. The value of $\sigma_{h,eq}$ according to Oding and cycles with R > 0 lies on the non-conservative side only for curves, which have n < 0.5. This is particularly valid for already mentioned 300M steels, where the greatest error leads to several tens of percent for n = 0.36. Thus the Oding's formula is very useful primarily for aluminum alloys, where the exponent *n* tends to the value of 0.5 generally. The asymmetry conversion according to (15) with exponent n = 0.4 must be adopted in the case of 300M steel, so that the originating error is optimized.

4. CONCLUSION

The proposed LESA method uses the results of linear stress fields from the FE-analysis and the common S-N curves to determine the fatigue damage. The principle of superposition is applied for damage calculation from more result files (i.e. from more load modes). Uniaxial methods with the conversion of three-dimensional stress states to equivalent uniaxial ones are used here. The method enables visualization of fatigue damages on FE-models leading to synoptic fatigue maps. Thus the critical areas can be revealed and any further damage analysis with higher precision can be done. To respect the multiaxial fatigue, the LPSA (local elastic-plastic stress and strain approaches) should be applied.

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